As a third example, suppose the section perpendicular to the long axis uniform in shape and area, we then get

$$\widehat{zz} = \frac{1}{2} \operatorname{R} \rho (c^2 - z^2),$$

$$\alpha/x = \beta/y = -\frac{1}{2} \operatorname{R} \rho \eta (c^2 - z^2)/E,$$

$$\gamma = \frac{1}{2} \operatorname{R} \rho z (c^2 - \frac{1}{3} z^2)/E$$

$$(79).$$

A special instance of this last case is presented by an elongated cylinder rotating about a perpendicular to its length through its centre. Putting $R = \omega^2$, we have

$$\alpha/x = \beta/y = -\frac{1}{2}\omega^{2}\rho\eta(c^{2}-z^{2})/E, \gamma = \frac{1}{2}\omega^{2}\rho z(c^{2}-\frac{1}{3}z^{2})/E$$
(80).*

Comparing the several cases of rotation, we have an interesting illustration of how the effects of the "centrifugal" force increase as the mean distance of the substance of the solid from the axis of rotation becomes larger. If the limiting angular velocity permissible in the elongated ellipsoid rotating about a short axis be taken as 100, then the limiting angular velocities in the thin elliptic disc and the elongated cylinder rotating about their short axes—the material, and the length of the long dimension being the same for all—are approximately 87 and 71 respectively, both on the stress-difference and greatest strain theories. A caution must, however, be added that in bodies of such elongated form rotating about a short axis, a sudden change in the angular velocity may prove disastrous.

"Micro-Metallography of Iron. Part I." By Thomas Andrews, F.R.S., M.Inst.C.E. Received December 15, 1894,—Read January 24, 1895.

Secondary Micro-crystalline Structure in Metallic Iron.

The term metallography appositely describes, in one word, that department of metallurgical science which deals with the accurate study and delineation of the ultimate formation or structure of metals, a knowledge of which is of the utmost importance.

This development of the science of metallurgy is destined to prove of incalculable value in the study of the ultimate micro-structure of iron and steel.

Experiments on the microscopic structure of iron and steel appear

* This is more exact for a long beam of rectangular cross section than the result I obtained in the 'Quarterly Journal' for 1888, p. 29.

to have been first initiated by Dr. H. Clifton Sorby, F.R.S., about thirty years ago, and the author about the same time commenced some investigations in this direction. Strange to say, the subject has since then lain almost dormant, and only recently have scientific investigators resumed the study of the ultimate microscopic structure of metals. Comparatively little progress has yet been made in this important and fertile field of research, and our knowledge of the true structure of metals is, consequently, at the present time limited. Excellent work in this direction is now, however, in progress at the hands of several investigators, and it is hoped that by means of further careful detailed observation with accurate modern microscopic appliances valuable and reliable results will accrue.

In the course of a research with high microscopical powers (including 300, 500, 800, 1200, and upwards to 2000 diameters) on the microcrystalline structure of large masses of wrought iron, the author had the privilege recently to observe the following novel metallurgical facts.

When large masses, several tons in weight, of practically pure wrought iron were allowed to slowly cool from a white heat, a secondary or subcrystallisation of the metallic iron occurred. The normal primary crystals of the iron, or those which have hitherto been regarded as constituting the ultimate structure of the metal, were found to inclose a subcrystalline formation consisting of very minute, and much smaller, crystals of pure iron also belonging to the regular order of crystallisation. These crystals sometimes manifested the hexagonal form, the predominant angle being about 120°, and often they assumed the form of simple cubes. The secondary crystals were contained within the area of the larger primary crystals.

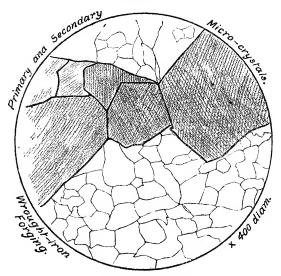
Typical illustrations of this duplex crystallisation found in two large iron forgings are given in figs. 1 and 2, and the relative dimensions of a number of individual crystals are given on Tables I and II.

The results of twenty measurements of the primary crystals and twenty measurements of the secondary crystals taken on each forging are given on these tables.

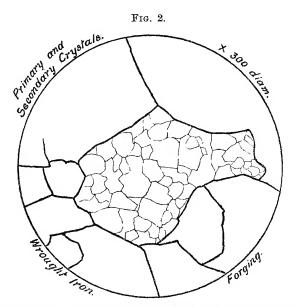
The markings of the intercrystalline spaces or junctions of the secondary crystals were very clearly defined, but they were exceedingly minute. The general form, contour, and relative size of the primary and secondary crystals, as seen in section, will be noticed on reference to the accurate tracings, figs. 1 and 2.

Judging roughly from the indications of the average micromeasurements on Tables I and II, there would appear to be approximately 1,000,000,000 of the secondary crystals in a cubic inch of the metallic iron.

Fig. 1.



Micro-section from large wrought-iron forging, showing primary and secondary micro-crystalline formation.



Micro-section from large wrought-iron forging, showing primary and secondary micro-crystalline formation.

Table I.—Secondary Micro-crystallisation of Metallic Iron.

Micro-section from large iron forging No. 1.				
Primary crystals.		Secondary crystals.		
Longitudinal dimensions of individual crystals in parts of an inch.	Transverse dimensions of individual crystals in parts of an inch.	Longitudinal dimensions of individual crystals in parts of an inch.	Transverse dimensions of individual crystals in parts of an inch.	
0·0273 0·0091 0·0091 0·0187 0·0050 0·0032 0·0077 0·0159 0·0227 0·0136 0·0136 0·0109 0·0068 0·0182 0·0054 0·0045 0·0045 0·0091 0·0118	0·0182 0·0059 0·0068 0·0114 0·0045 0·0027 0·0073 0·0136 0·0159 0·0136 0·0114 0·0109 0·0050 0·0159 0·0045 0·0045 0·0045 0·0045 0·0045	0 · 0012 0 · 0014 0 · 0016 0 · 0008 0 · 0010 0 · 0018 0 · 0012 0 · 0012 0 · 0010 0 · 0008 0 · 0020 0 · 0006 0 · 0016 0 · 0008 0 · 0020 0 · 0012 0 · 0008	0.0008 0.0008 0.0008 0.0016 0.0008 0.0016 0.0008 0.0010 0.0008 0.0010 0.0006 0.0020 0.0012 0.0006 0.0020 0.0020 0.0020 0.0020 0.0020 0.0020 0.0020	
Average 0:0112	Average 0.0090	Average 0 :0012	Average 0 0010	

In the case of both the primary and secondary crystals the predominant well-defined angles of the facets of the crystals hovered more or less about the angle of 120°. The majority of the angle readings, made with the goniometer attached to the microscope, indicated generally a hexagonal structure or form of crystallisation. There were, however, also perfect cubical crystals observed.

The observations were made with a Ross' first-class microscope. The micro-measurements afford an indication of the comparative size of the primary and secondary crystals. These measurements were carefully taken by a Jackson micrometer and in some cases by a Ramsden screw micrometer, both accurately calibrated with a standard stage micrometer. The wrought-iron forgings on which the observations were made were constituted of practically pure hammered wrought iron, the dimensions of the mass being about 10 feet long and about 12 inches square. The great length of time required for

such large masses of iron to cool from a white heat appeared to facilitate the production of the crystals of the secondary formation.

Table II.—Secondary Micro-crystallisation of Metallic Iron.

Micro-section from large iron forging No. 2.					
Primary crystals.		Secondary crystals.			
Longitudinal	Transverse dimensions of individual crystals in parts of an inch.	Longitudinal	Transverse		
dimensions of		dimensions of	dimensions of		
individual		individual	individual		
crystals in parts		crystals in parts	crystals in part		
of an inch.		of an inch.	of an inch.		
0 ·0149	0 · 0100	0 ·00100	0.00066		
0 ·0060	0 · 0050	0 ·00050	0.00024		
0 ·0060	0 · 0030	0 ·00050	0.00050		
0 ·0180	0 · 0120	0 ·00066	0.00050		
0·0160	0 · 0080	0.00100	0 ·00083		
0·0090	0 · 0048	0.00083	0 ·00083		
0·0060	0 · 0052	0.00050	0 ·00050		
0·0084	0 · 0060	0.00033	0 ·00033		
0 ·0060	0 ·0030	0.0022	0 · 0016		
0 ·0090	0 ·0070	0.0010	0 · 0008		
0 ·0100	0 ·0060	0.0016	0 · 0012		
0 ·0050	0 ·0044	0.0010	0 · 0008		
0 ·0050	0 · 0040	0·0010	0 ·0008		
0 ·0100	0 · 0050	0·0004	0 ·0004		
0 ·0048	0 · 0046	0·0012	0 ·0012		
0 ·0072	0 · 0054	0·0008	0 0008		
0·0080 0·0056 0·0110 0·0080	0·0060 0·0016 0·0070 0·0060	0.0010 0.0010 0.0010	0 0008 0 0008 0 0008 0 0006		
Average	Average	Average	Average		
0.0086	0:0057	0 ·00097	0.00075		

In the case of both the primary and secondary crystals the predominant well-defined angles of the facets of the crystals hovered more or less about the angle of 120°. The majority of the angle readings, made with the goniometer attached to the microscope, indicated generally a hexagonal structure or form of crystallisation. There were, however, also perfect cubical crystals observed.

The rationale of this duplex crystallisation has apparently been as follows:—The mass of metallic iron on cooling having reached the crystallising point at about 740° C., the periphery or skeletons of the larger or primary crystals were then formed. As the period of cooling was, however, very slow, the semi-fluid or viscous metal in the interior of these primary crystals was, on finally consolidating,

apparently further broken up or subdivided into a considerable number of smaller crystals, inclosed within the boundary or periphery of the primary crystals.

In the course of further experiments on the cooling of large masses of wrought iron, the author has also found, by the use of high power objectives, that the secondary crystals sometimes inclosed a still more minute form of crystals of pure iron, of the cubical form, which may hence be regarded as constituting a tertiary system of crystallisation in pure metallic iron. These experiments therefore indicate that large masses of heated wrought iron, on cooling from above the temperature of the crystallisation of metallic iron, viz., 740° C..* are capable of crystallising in three distinct modifications, which may tentatively be called the primary, secondary, and tertiary system of crystallisation in iron, these various crystalline modifications being all, however, connected with the regular system of crystallisation. The author has microscopically examined numerous large masses of practically pure wrought iron varying in weight from about 2 tons to 4 cwt., and even less, and he finds these subcrystalline formations to be frequently present, consequent on the slow cooling of such large masses.

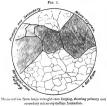
The crystals of this secondary formation are not often distinctly discernible in smaller masses of metallic iron, such as rolled rods, plates, or sheets, as these in the course of manufacture rapidly cool, and are frequently manipulated during the finishing processes at temperatures below the crystallising point of wrought iron (740° C.). The author has, however, observed the presence of this subcrystalline structure in small masses of iron, but, in these instances, the subcrystals are generally smaller in size and not always so distinctly marked as those found in larger masses of metallic iron.

The microscopical examinations were made on carefully-prepared and polished samples, etched in nitric acid (1 part HNO₃, sp. gr. 1·20, and 49 parts water), and by the use of high microscopical powers ($\frac{1}{6}$ inch to $\frac{1}{10}$ inch, and other objectives). The drawings were accurately made with the camera Lucida.

In each observation the etching was prolonged, under constant observation with lenses, a suitable time to develop the accurate structure of the metal.

The varied forms of crystallisation observed in this research and referred to in this paper appear to be of such novel metallurgical interest, that the author felt it desirable at once to record the observations, and he hopes to be able to furnish the results of further investigations in this direction.

^{*} The temperature at which pure iron crystallises, viz., 740° C., has recently been approximately determined with great care and accuracy by Professor J. O. Arnold, F.C.S., at the Sheffield Technical School.





Micro-section from large wrought-iron forging, showing primary and secondary morro-crystalline formation.